# A Formation Flying Strategy for CloudSat/Picasso-Cena[1]

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*Abstract*—For CloudSat and Picasso/Cena to collect nearly simultaneous data over the same territory CloudSat's ground track must stay within +/- 1 km (in cross-track) of Picasso/Cena's. The Department of Defense Space Test Program (STP) is responsible for CloudSat mission operations and its contractors must determine and execute orbit maneuvers approximately once per week to maintain the ground track separation. This paper derives an optimal path for CloudSat relative to Picasso that minimizes the average cross-track separation. Using historical atmospheric data, the paper then shows that simple parabolic fits of the satellites' relative locations, determined from GPS data, can be used to calculate the size and timing of the maneuvers. With a judicious selection of the relative location thresholds to trigger the maneuvers, this simple approach meets the cross-track and maneuver frequency requirements. Furthermore, this approach does not require highly accurate orbit propagation or atmospheric density models.

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## 1. INTRODUCTION

CloudSat and Picasso/Cena are a pair of cooperative NASA missions to investigate how clouds affect climate and to improve weather prediction models. CloudSat will carry a 94-GHz radar and Picasso/Cena will carry a lidar. In order for these spacecraft to collect nearly simultaneous data over the same region of the earth, CloudSat must fly in formation relative to Picasso. CloudSat must keep its ground track within +/- 1km (in cross-track) of the ground track of Picasso/Cena with at most a 15 second time separation. They were originally scheduled to be launched together in 2003, but it now appears that Picasso/Cena may be launched up to a year after CloudSat and CloudSat will fly a similar formation with another satellite, Aqua, while retaining the goal of flying in formation with Picasso/Cena when it reaches orbit.

The Department of Defense Space Test Program (STP) has hired contractors at the RDT&E Support Complex (RSC) at Kirtland Air Force Base to conduct CloudSat mission operations. It is the responsibility of operators at the RSC to maintain the +/- 1 km cross-track maximum separation of the ground tracks. They will periodically command CloudSat to execute in-track maneuvers to lower its orbit, and thus speed it up, before it falls too far behind Picasso/Cena. It is desirable that these maneuvers occur no more often than about once a week on average in order to limit their impact on data collection. The use of 'braking' maneuvers to avoid overshooting should be limited to avoid significant impact on fuel consumption.

Accurately predicting a satellite's absolute location several days in advance requires very accurate orbit propagation programs with detailed models of the earth's gravity, the changing atmosphere, and other perturbing effects. There was initial concern that performing this task would require upgrades to the current RSC capabilities and put a significant burden on the operators and orbit analysts. However, it is the satellites' relative, not absolute, locations that are needed. Data from the on-board GPS receivers provide an easy way to determine the satellites' relative motion over a period of days prior to a maneuver and include an implicit estimate of the atmospheric density. It was the purpose of this study to demonstrate that, with a simple parabolic model of the relative motion, the maneuvers could be easily determined from a very limited subset of the GPS data and would meet the requirements even during periods of wide fluctuations in the atmospheric density. This approach similar to one using a linaer model of the semi-major axis decay described in [1].

After initial maneuvering, the satellites will be in nearly circular, highly inclined orbits at an altitude of about 700km. CloudSat trails Picasso/Cena and CloudSat's orbit is rotated slightly in right ascension to compensate for the earth's rotation. Picasso/Cena will have a lower ballistic coefficient and, therefore, the orbit of Picasso/Cena will decay more rapidly. As its total energy decreases, Picasso/Cena's velocity will increase and its period will decrease compared to those of CloudSat. Left unchecked this would cause CloudSat to fall farther behind Picasso/Cena and its ground track would keep moving farther to the west relative to Picasso/Cena's ground track.

[1] 0-7803-6599-2/01/$10.00 © 2001 IEEE

Eventually, the ground tracks would be separated by more than a kilometer for at least part of their orbits. After a maneuver CloudSat will be lower and gaining on Picasso/Cena. When Picasso/Cena's orbit decays to the same altitude as CloudSat the satellites will be moving at the same speed. When Picasso/Cena falls below CloudSat again, CloudSat will 'turn-around' and begin to fall back relative to Picasso/Cena.

Section 2 will view CloudSat's in-track motion relative to an 'ideal' location on its orbit. That is the point in CloudSat's orbit where CloudSat's ground track would match Picasso/Cena's. We'll relate this relative in-track motion to the cross-track separation at the equator. We'll describe the selection of an optimal 'turn-around' point relative to this 'ideal' location. Under idealized conditions, resulting in a constant relative acceleration, the relative motion is simply parabolic and optimal 'turn-around' point minimizes the average cross-track separation.

However, as we'll see in Section 3, the atmospheric density (and thus the relative acceleration) at this altitude can change by a factor of two or more over the course of a few days. This is the primary source of error when determining the size of the relative velocity change required to achieve a desired 'turn-around' point. There will also be uncertainties in the location of the satellites as determined by on-board GPS receivers. These will affect the actual 'turn-around' location to a lesser extent. It is also necessary to account for delays between the collection of the GPS data and the actual time of the maneuvers. This section finishes with a suggested relative location to conduct the maneuver and a desired near-optimal 'turn-around' point that allows ample margin for the atmospheric variation and other uncertainties, while maintaining the desired maneuver frequency.

The parabolic motion model can then used to estimate the time and desired relative velocity change of a maneuver, including a braking maneuver, if necessary. This approach is described in section 4. The results of studies that included a realistic, though somewhat coarse, atmospheric model, expected errors in GPS and errors in thruster output indicate this strategy will maintain the cross-track requirement even through the worst periods of atmospheric density variation.

Sections 5 and 6 discuss the validity of the simplifying assumptions and additional simulation using higher fidelity models to confirm these results.

## 2. Idealized Relative Motion

Figure 1 illustrates the orbit and ground track geometry of CloudSat relative to Picasso/Cena at the point Picasso/Cena is crossing the equator from North to South (descending). In

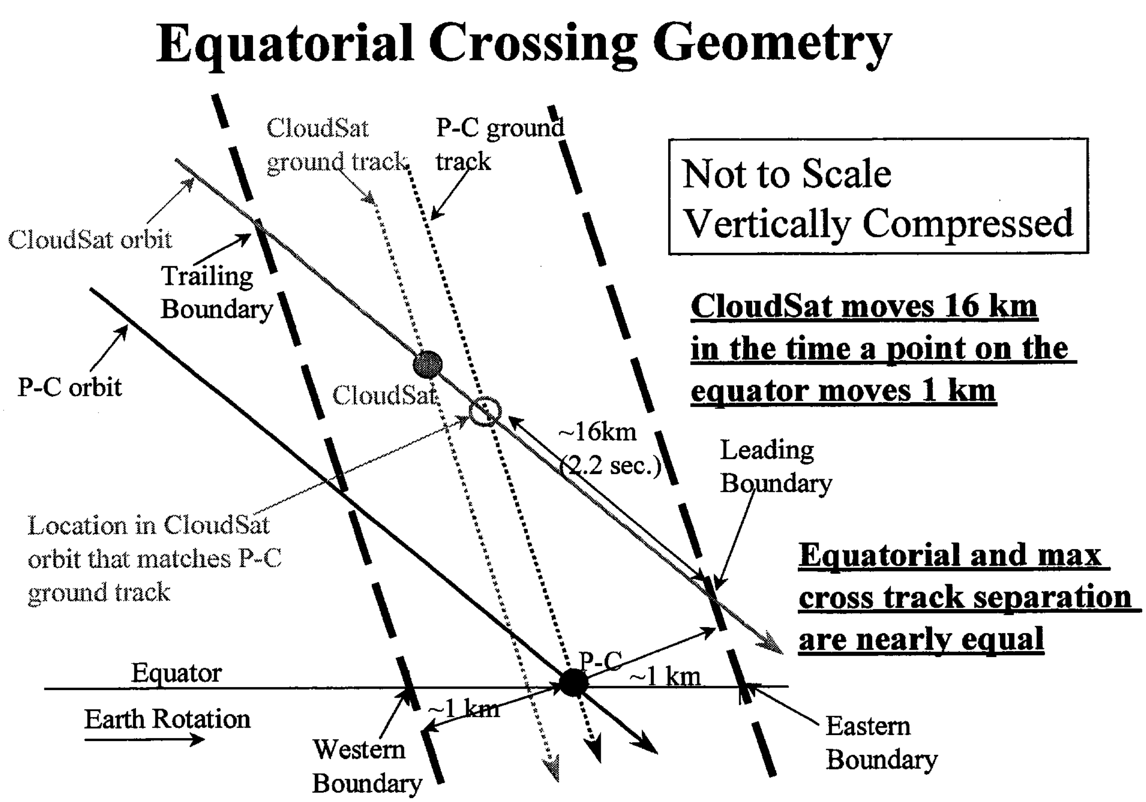



**Figure 1**

order to limit the chance of the satellites colliding in the event of a loss of control, CloudSat trails Picasso/Cena. The descending node of CloudSat is offset about .06° to compensate for the earth rotation in the time between Picasso/Cena's equatorial crossing and CloudSat's equatorial crossing. At the time Picasso/Cena is crossing the equator there is an 'ideal' location in the CloudSat orbit. If CloudSat is at that 'ideal' location it will cross the equator at the same longitude as Picasso and if their orbits are identical except for the right ascension offset their ground tracks would be entirely matched. An 'ideal' location could be determined for any latitude, but the equator will be most useful for this analysis.

Most of the analysis in the remainder of this paper will be based on CloudSat's distance from the 'ideal' point measured once per orbit when Picasso/Cena is crossing the equator at its descending node.

The initial orbit for the satellites is nearly circular (eccentricity .0001179) with a semi-major axis of 7077 km (altitude about 700 km) and an inclination of 98°. The inclination and eccentricity were chosen to maintain a 'frozen' argument of perigee at -90°. This orbit results in about 14.5 orbits per sidereal day. So, CloudSat's ground subpoint is moving about 14.5 times as fast as a point on the equator, or 6.74 km/sec. CloudSat itself is traveling on a 700 km larger circle than the subpoint, and travels about 11% faster. It is moving about 16.1 times as fast as an equatorial ground point or 7.49 km/sec.

To simplify the analysis and presentation, let's assume CloudSat is moving 16 times as fast as an equatorial point. **Then, if CloudSat is within 16 km of the 'ideal' point when Picasso/Cena is crossing the equator, CloudSat will cross the equator within 1 km of Picasso/Cena's equatorial crossing point.**

The requirement is to keep the cross-track separation of the ground track within +/-1 km throughout the orbit. **However, for the nearly circular orbits we are considering it is sufficient to keep the equatorial crossing points within +/- 1 km, or equivalently to keep CloudSat within +/- 16 km of its 'ideal' point when Picasso is at the equator.** There are two reasons for this. First, if CloudSat's crossing point at any latitude (including the equator) is within 1 km of the point where Picasso crossed that latitude, then the cross-track distance between the ground tracks at that latitude is less than 1 km. This is because the cross-track is the shortest distance between the (nearly) parallel ground tracks. With CloudSat's inclination the cross-track distance is about 1% less at the equator. At higher latitudes this effect is even larger since the angle between the ground track and the latitudinal lines decreases. Second, the speed of a ground point rotating with the earth decreases with latitude as the cosine of the latitude. At 60 degrees latitude it takes twice as long for a point on the earth to rotate 1 km as is does at the equator. So at this latitude CloudSat only needs to be within +/- 32 km of its 'ideal' location to cross within +/- 1 km of the point at which Picasso crossed the latitude. The differences in the orbits of Picasso/Cena and CloudSat will cause the distance between CloudSat and its 'ideal' point to change slightly during an orbit, but not enough to overcome the 1/cos(latitude) expansion factor of CloudSat's acceptable range. This will be discussed further in section 5.

So, the +/1 km range of acceptable equatorial crossing points can also be thought of as a +/- 16 km in-track range for CloudSat relative to the 'ideal' location. The eastern equatorial boundary point corresponds to the leading in-track boundary point and the western equatorial boundary point corresponds to the trailing in-track boundary point.

Figure 2 shows the correspondence of the in-track location of CloudSat relative to its 'ideal' location and the difference in the equatorial crossings of the satellites.

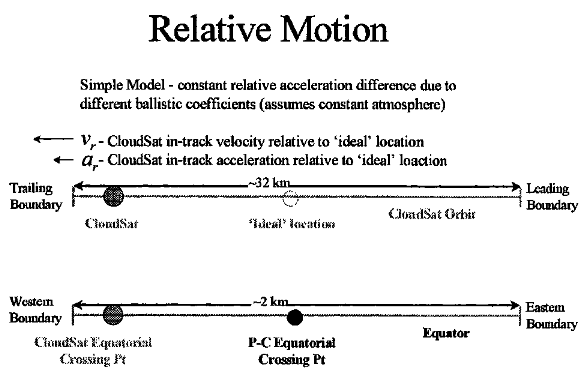



**Figure 2**

For now, let's make the idealized assumption that the motion of CloudSat relative to Picasso/Cena and, therefore, relative to the 'ideal' point, is due only to an acceleration difference, $a_{diff}$, caused by the difference in their ballistic coefficients (estimated to be 42.5 kg/m**2 for CloudSat and 26 to 30 kg/m**2 for Picasso/Cena). This means we are assuming the two satellites have the same inclination, eccentricity, and argument of perigee, which they very nearly do. Let's further assume for the moment that the atmospheric density is constant so $a_{diff}$, is constant (atmospheric density variation will be considered in sections 3 and 4). So over the course of an orbit CloudSat would experience a difference in $\Delta V$

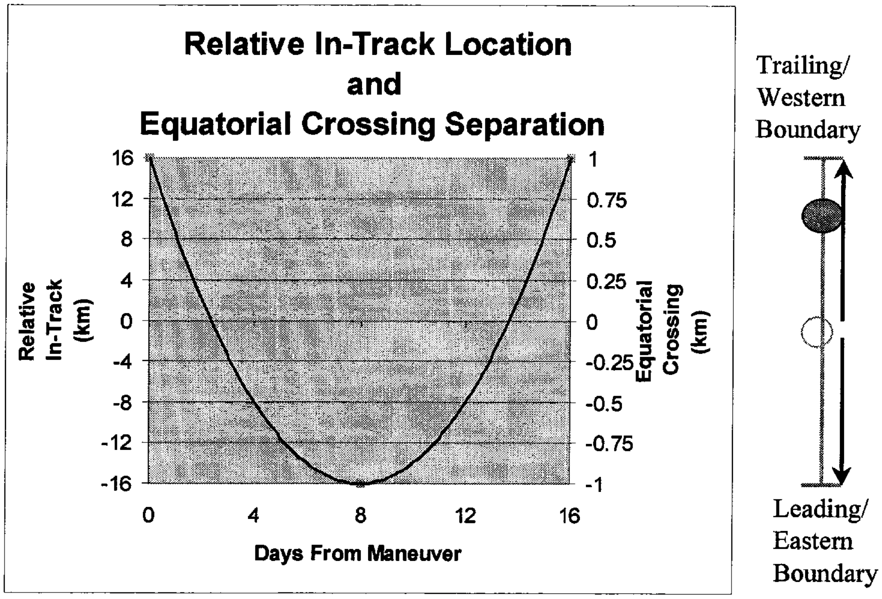
$$\Delta V_{diff} = P * a_{diff} \tag{1}$$

where $P$ is the orbit period.

Since a small $\Delta V$ in the anti-velocity direction actually results in an increase in the satellite's speed by an amount $\Delta V$ [2] the satellite experiencing the greater acceleration (Picasso/Cena) will be moving faster.

For near circular orbits the drift rate of a satellite relative to its initial position for a small $\Delta V$ is $1080 * \Delta V / V$ degrees per orbit and in the opposite direction of the $\Delta V$.[3] This is

# Parabolic motion




Simple Model 16 day Boundary to Boundary Round Trip

**Figure 3**

because the satellite is not only moving faster, but is traveling in an orbit with a smaller radius. Converting the angles to distances, after an orbit the satellite is traveling at a velocity $3*\Delta V$ relative to its initial position. If CloudSat starts at its 'ideal' location (which remains fixed relative to Picasso/Cena's position) CloudSat is moving at a speed $3*\Delta V_{diff}$ relative to the 'ideal' point in the opposite direction of the acceleration difference. Therefore, relative to the 'ideal' point CloudSat moves as though it is under a constant relative acceleration

$$a_r = 3*a_{diff} \qquad (2)$$

in the direction opposite CloudSat's velocity vector.

Under this constant acceleration assumption CloudSat's motion relative to the 'ideal' point is a simple parabola, the path of any object under a constant acceleration. Figure 3 shows an example of the in-track motion of CloudSat relative to the 'ideal' point due to a constant relative acceleration, $a_r = 1$ km/day**2. The equatorial crossing differences are 1/16 of the corresponding relative in-track locations and are therefore separating with an acceleration of $a_{eqx} = 1/16$ km/day**2.

Notice, however, that during the boundary-to-boundary round trip half the time (days 4 through 12) is spent with a cross-track separation at the equator of greater than .5 km (in-track distance from the 'ideal' point greater than 8 km). Figure 4 shows a family of potential paths of CloudSat's motion relative to the 'ideal' point under a relative acceleration, $a_r = 1$ km/day**2. Each path corresponds to a maneuver conducted at the trailing in-track boundary but with a different distance from the 'ideal' point at the time CloudSat 'turns around'. Given a relative acceleration, $a_r$, and a desired distance from the 'ideal' point at 'turn-around', $d_T$, the required relative velocity, $v_r$, immediately following the maneuver is

$$v_r = \sqrt{2*a_r*(16-d_T)} \qquad (3)$$

The round trip time, $T_R$, for CloudSat to return to the trailing boundary is

$$T_R = 2*\sqrt{2*(16-d_T)/a_r} \qquad (4)$$

If $d(t)$ is the relative location at a time $t$ following the maneuver, then

## Possible Paths

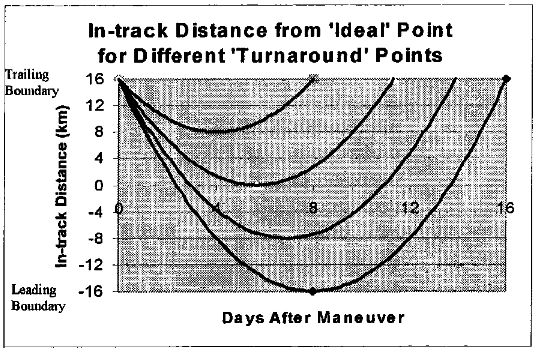



**Figure 4**

## Optimum 'Turnaround'

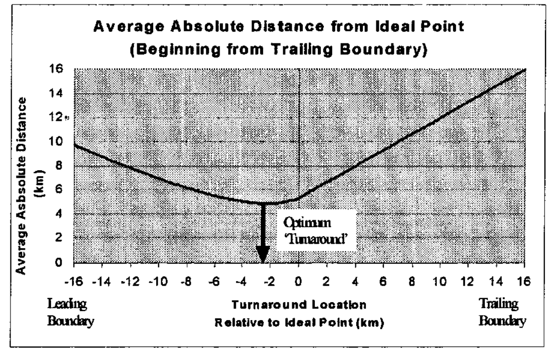



**Figure 5**

$$d(t) = 16 + v_r * t + a_r * t^2 \quad (5)$$

For a given 'turn-around' point, $d_T$, the average absolute distance $|d(t)|$ from the 'ideal' point is given by

$$(16 + 2 * d_T)/3 \text{ for } d_T \geq 0$$

and (6)

$$\{2 * d_T + 16 - 4 * d_T \sqrt{-d_T /(16 - d_T)}\}/3 \text{ for } d_T < 0$$

It is interesting to note that this average is independent of the acceleration. Figure 5 is a graph of this average as a function of $d_T$. The minimum average value of –4.9 km occurs at $d_T \approx -2.2$ km. See the appendix for derivations of equations (3), (4), and (6).

So, if you start at the trailing boundary, a maneuver strategy that minimizes the average distance from the 'ideal' point over the round trip, and thus minimizes the average equatorial separation, would aim for a 'turn-around' 2.2 km beyond the 'ideal' point. This still leaves a 76% (13.8/18.6) margin beyond the desired 'turn-around' point to the leading boundary. As we'll see, this margin will be useful to accommodate uncertainties in the relative position and velocity of the satellites and the uncertainty in the relative acceleration due to changes in the atmospheric density. Also, since the round trip time is proportional to the square root of the distance between the maneuver and the 'turn-around' point the round trip time would still be about 75% ($\sqrt{18.2/32}$) of the 16 day boundary to boundary round trip or approximately 12 days. So, while aggressively aiming for the leading boundary would maximize round trip time, aiming for a point well back from the boundary improves the average separation and reduces the risk of overshoot.

If a different point is chosen to conduct the maneuvers, the distances scale and the ratio of the distance from the maneuver location to the 'ideal' location and the 'ideal' location to the 'turn-around' point is still about 7 to 1. So, the optimum 'turn-around' is then approximately $d_m/7$ beyond the 'ideal' point, where $d_m$ is the distance from the maneuver point to the 'ideal' point.

### 3. CHOOSING A MANEUVER STRATEGY

Under the ideal conditions of constant acceleration, $a_r$, and perfect knowledge of the relative positions and velocity of the satellites, the selection of a maneuver strategy would be clear. First, decide how long you want for a round trip time between maneuvers, $T_R$. Then in equation (4) the distance, $d$, from the maneuver to the 'turn-around' point replaces $(16 - d_T)$ and $d = T_R^2 * a_r / 8$. To minimize the average distance from the 'ideal' point for this maneuver frequency, conduct the maneuvers when CloudSat's in-track location relative to the 'ideal' point is at $7 * d/8$ and choose the desired relative velocity, $v_0$, after the maneuver so that the 'turn-around' occurs at $-d/8$. Since the relative velocity goes from $v_0$ to $-v_0$ in the time $T_R$ under constant acceleration, $a_r$,

$$v_0 = T_R * a_0 / 2 \quad (7)$$

Under the nominal in-track acceleration of 1 km/day**2 used in section 2, an 8 day round-trip would mean a maneuver point 7 km from the 'ideal' in-track location toward the trailing boundary and a 'turn-around' point 1 km beyond the 'ideal' location toward the leading boundary. This would keep the average separation small while requiring a maneuver frequency of about a once per week.

However, the atmospheric density is not constant and while the GPS data is quite accurate (9 meters, 1-sigma, with selective availability off) it does contribute to uncertainty in the equatorial crossing separation both directly and through errors in the estimation of relative velocity and acceleration between the spacecraft. Some filtering could improve the accuracy of the GPS data, but to be conservative in this analysis we've assumed little ground processing of the GPS data is required. In addition, one cannot instantaneously command CloudSat to conduct a maneuver based on current information. The GPS data from Picasso/CENA may be a day or more old when it is received at the RSC and it may take the better part of a day before the data is processed and the maneuver commands are uploaded to CloudSat and executed. So the data on which the maneuver is determined may be up to two days old by the time the maneuver is executed.

Figure 6 is a plot of the Mass Spectrometer Incoherent Scatter (MSIS) model of atmospheric density (based on measured data) for four two-year segments corresponding to the two-year portion of the solar cycle during which this mission is planned. Variations in density of a factor of two or more over the course of a few days is not uncommon. While there is some ability to predict the atmospheric density a day or so into the future there is no reliable way to predict the timing of the large variations associated with solar storms. Therefore, any maneuver strategy needs to provide enough margin to account for an unexpected drop or increase in atmospheric density. It should rarely result in an overshoot of the leading/eastern boundary. In the case of a potential overshoot it should allow enough time to recognize the problem after the maneuver and conduct a 'braking' maneuver before the boundary is reached.

Given an initial relative velocity, $v_0$, and a constant relative acceleration, $a_r$, the distance between the maneuver and the 'turn-around point' is $v_0^2/(2*a_r)$. So an overestimate of the actual relative acceleration by a factor of two would result in a 'turn-around point' twice as far from the maneuver point as expected. To accommodate the unpredictability of the atmosphere it would seem prudent to plan for at least a factor of two overestimate of the atmospheric density. To begin, let's allow for a maneuver in which the change in relative location from the maneuver point to the 'turn-around' point is three times larger than expected. Assuming a nominal relative acceleration of 1 km/day**2, selecting a maneuver point 8 km from the 'ideal' in-track location (toward the trailing boundary) and a 'turn-around' at the 'ideal' in-track location looks promising in several ways;

1) the nominal time between maneuvers would be 8 days,

2) there is a 200% margin for overshoot toward the leading boundary,

3) there is 100% margin toward the trailing boundary to

## Atmospheric Density at 700 km

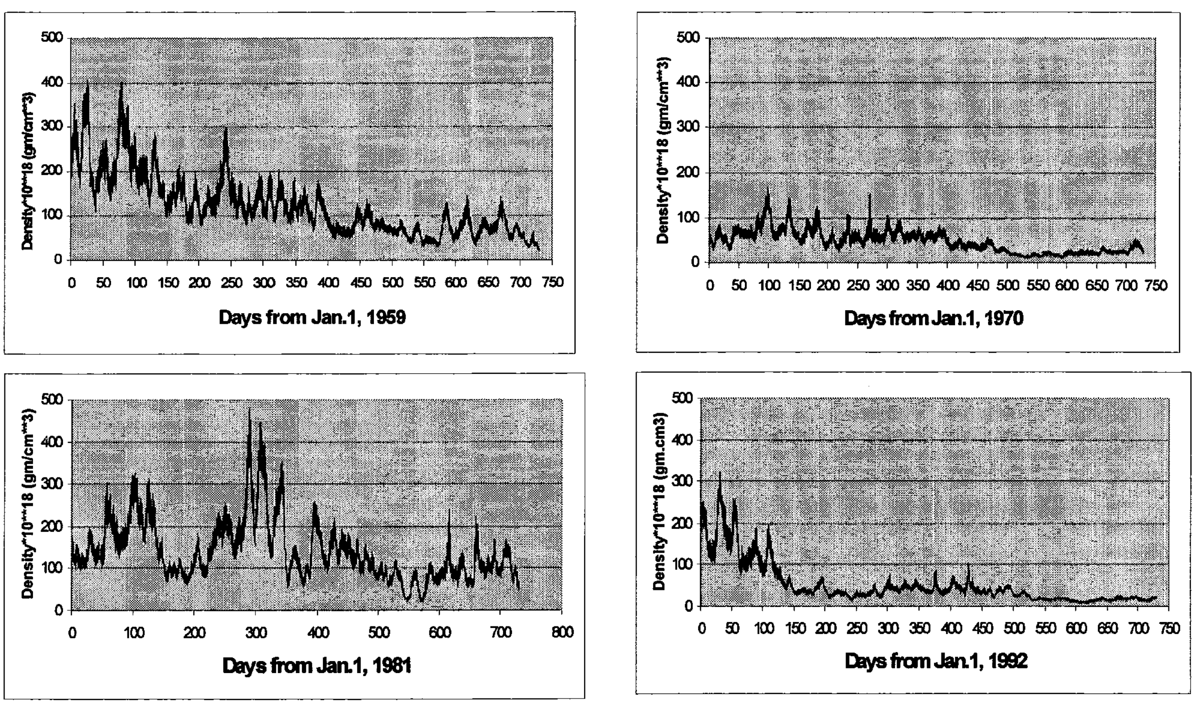



**Figure 6**

accommodate for atmospheric uncertainty in the two days between the time the data is collected and the maneuver is calculated and any delay in executing the maneuver.

4) the 'turn-around' point in near optimal to minimize the average separation for the nominal atmosphere

## 4. CALCULATING/MODELING THE MANEUVERS

How much does the changing atmospheric density influence the relative motion between the satellites? Allowing two days from the collection of GPS data to the execution of a maneuver and the desire to conduct maneuvers on average no more frequently than once a week, can a strategy like that proposed in the previous section maintain the relative locations within the required limits under a wide range of atmospheric conditions?

For initial modeling purposes we'll assume that the relative acceleration between the satellites is proportional to the atmospheric density. Slight differences in the eccentricities and arguments of perigee will cause some relative motion on top of this acceleration, but this will average out over the course of an orbit. Since the allowable in-track separation increases as 1/cos(latitude), the maneuver algorithm can focus on the equatorial crossings as long as this additional motion is not enough to cause the in-track separation limit to be exceeded at a latitude other than equator before it is exceeded at the equator. One must consider differences between the ascending and descending equatorial crossings, as will be discussed in section 5.

Using the MSIS data the relative motion between the satellites can then be modeled by converting the atmospheric density to a relative acceleration and numerically integrating to obtain relative velocity and position as a function of time.
The interval between data points in the MSIS data is 3 hrs, which is short compared to the scale of atmospheric variation and maneuver intervals. So, treating the relative acceleration between the satellites as an average represented by each 3 hour data point is a reasonable assumption.

A conversion factor is needed to transform the atmospheric density into a relative acceleration. For near circular orbits the change in velocity each revolution is

$$\Delta V_{rev} = \pi * \rho * s * V * B^{-1} \tag{8}$$

where $\rho$ is the atmospheric density, $s$ is the semi-major axis, $V$ is the satellite's velocity, and $B$ is the satellite's ballistic coefficient[4]. Since CloudSat and Picasso/Cena are in nearly identical orbits, only the difference in their ballistic coefficients causes their velocities to change differently. So for each revolution the difference in their velocity change is

$$\Delta V_{diff} = \pi * \rho * s * V * (B_{PC}^{-1} - B_{CS}^{-1}) \tag{9}$$

where $B_{PC}$ and $B_{CS}$ are the ballistic coefficients of Picasso/Cena and CloudSat, respectively.

The ballistic coefficients of CloudSat and Picasso/Cena are still uncertain, but CloudSat's current estimate is about 42.5 kg/m**2 and Picasso/Cena's is between 26 and 30 kg/m**2. Using $B_{CS} = 42.5$, $B_{PC} = 27.6$, $\rho = 125 * 10^{-18}$ g/cm**3, and the nominal orbit parameters, $s = 7077$ km, and $V = 6.49 * 10^5$ km/day and converting the $\Delta V_{diff}$ from km/day/rev to km/day/day gives a difference in their velocity change, in other words an acceleration difference, $a_{diff} = .333$ km/day**2. Then, from equation (2), $a_r = 3 * a_{diff} = 1$ km/day**2. Since $\rho = 125 * 10^{-18}$ g/cm**3 is a representative atmospheric density from the graphs in Figure 6, this establishes 1 km/day**2 relative acceleration as a reasonable reference value. With these

# Maneuver Strategy

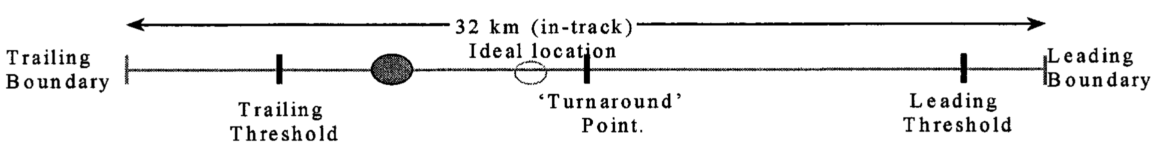



Figure 7

ballistic coefficients then the atmospheric density can be converted to relative acceleration by

$$a_r = \rho/(125*10^{-18}) \qquad (10)$$

where $a_r$ is in km/day**2 and $\rho$ is in g/cm**3.

With this conversion factor the MSIS data was converted into relative accelerations. This established a set of representative relative accelerations that could be used to test the maneuver strategy described in section 3. It was relatively simple to model the relative motion and include the variation in the atmosphere, the time delay between collection of the GPS data and the maneuver, the uncertainty in the GPS data, and the uncertainty of the thrust during the maneuvers.

To implement a strategy we must pick a relative location to be the trailing threshold at which to conduct the 'standard' (non-braking) maneuvers and a desired 'turn-around' point. Also, we need to pick a leading threshold to conduct braking maneuvers, if necessary. These points are shown in Figure 7.

The procedure is as follows:

1) Determine the 'ideal' time difference between the equatorial crossings of Picasso/Cena and CloudSat. This can be determined from the difference in their right ascensions or from the longitudes of their crossings (from the GPS data) using the rotation rate of the Earth.

   Since the relative ascending nodes of the satellites and their velocities will not change appreciably between maneuvers, this would only need to be calculated following each maneuver. Also, an error in this 'ideal' time difference only affects the 'ideal' point. It does not affect the relative velocity and acceleration calculations. These are more critical since errors in velocity and acceleration propagate throughout the time between maneuvers.

2) Estimate the in-track distance of CloudSat from its 'ideal' point. This should be done at the same place in the orbit so that minor differences in the orbital parameters don't affect the calculation of the dynamics due to atmospheric variation. Either the ascending or descending crossings of the equator by one of the satellites is an obvious choice.

   One can determine the actual crossing time difference by interpolation from GPS locations before and after the equator crossings. Convert this time difference to a location relative to the 'ideal' point by subtracting the 'ideal' time difference to get a relative time difference between the actual and 'ideal' equatorial crossing. Multiply this by CloudSat's actual (inertial) velocity or interpolate the GPS data to get the distance between two locations separated by the relative time difference. To be perfectly correct, these locations should be converted to inertial space but the impact of the earth rotation is small for this inclination and orbit velocity.

3) Estimate the relative velocity and acceleration by fitting the relative locations with a parabola.

4) Project the relative location forward two days (to account for the time between the data collection and the maneuver) using the estimated relative acceleration and velocity. Check if the in-track distance will exceed either threshold.

5) If a threshold will be exceeded, calculate a desired change in relative velocity, to be executed at the estimated time that the threshold will be crossed.

   a) In the case of a standard maneuver (approaching trailing/western boundary), the change in relative velocity is designed to achieve desired 'turn-around' point assuming parabolic motion using the estimated acceleration.

   b) In the case of a braking maneuver (approaching the leading/eastern boundary), the relative velocity change should just stop the relative motion so CloudSat will start to drift back toward the trailing boundary.

6) Execute that maneuver at the appointed time and re-start the process.

The exact point to conduct a maneuver would be chosen so that the maneuver's location within the orbit would best maintain CloudSat's orbital elements relative to Picasso's. This is still being studied and is discussed briefly in section 6.

Except for step 1, this whole process was simply modeled in a spreadsheet. As mentioned previously, the 'ideal' time difference or location is not a major contributor to errors in the maneuver determination since it does not affect the relative velocity and acceleration calculations. Starting with an initial relative location and velocity, 'true' in-track relative velocities and locations were created by numerically integrating the relative acceleration based on the atmospheric density data. As maneuvers are generated they are added to 'true' relative velocity calculation (including any adjustment for uncertainty in the actual thrust).

# Maneuvers without Braking

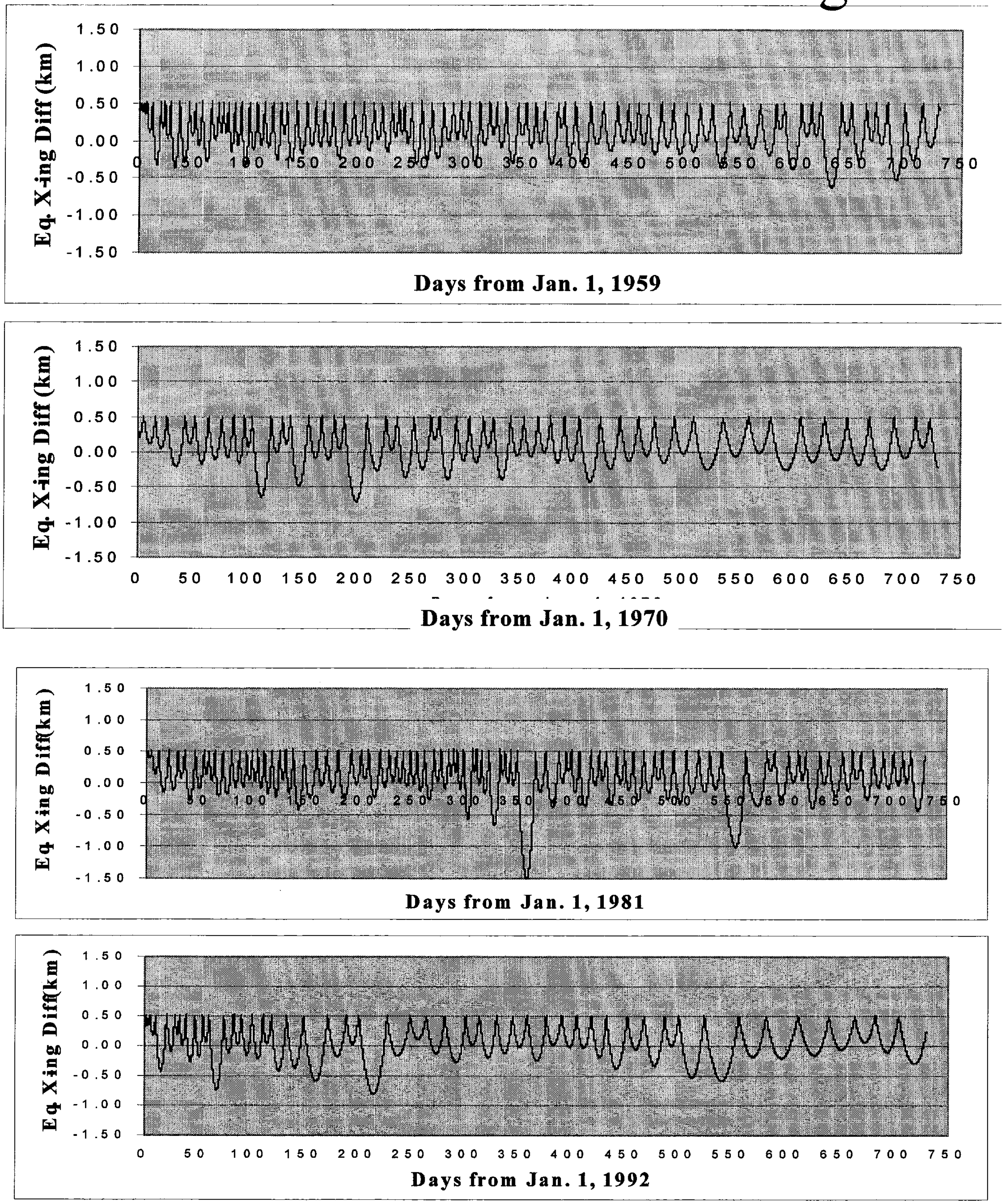



**Figure 8**

# Overshoot Due to Density Drop

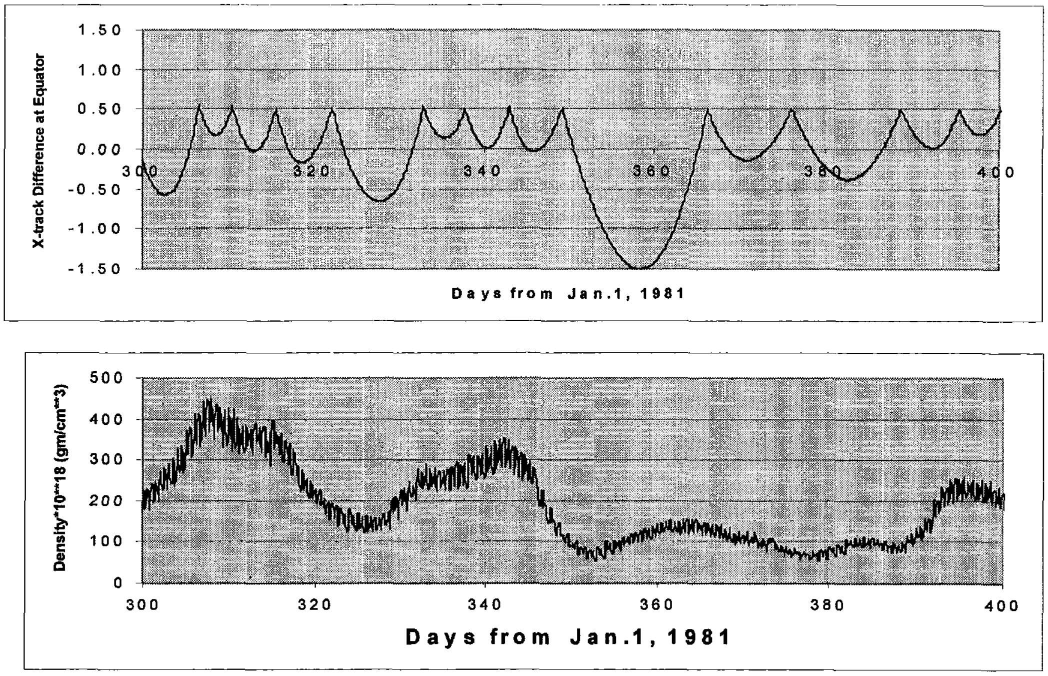



**Figure 9**

Initially, the process was modeled without errors in the relative positions or thrusts and without a leading threshold. The acceleration estimate used for computing maneuvers was the average 'true' relative acceleration during the day ending two days before the maneuver. This was done to determine how often a maneuver would overshoot the leading/western boundary due to changes in the atmospheric density. This would give an initial feeling of how large a margin would be needed since atmospheric variation is the major error source. Because of the nature of the MSİS data, these relative locations, velocities and accelerations were generated at 3-hour intervals rather than at the equatorial crossings. This is appropriate for this model since we are just considering the in-track relative motion due to the atmospheric drag.

Figure 8 shows the equatorial crossing separations for each of the four two-year periods using the 8 km relative in-track (.5 km equatorial) threshold for standard maneuvers and a 0 km relative in-track (0 km equatorial) desired turn-around. This threshold and the 'turn-around' point were selected through some trial and error. The results vary with initial conditions since that determines the timing of the maneuvers relative to the sharp drops in atmospheric density that occasionally occur. The results shown are among the 'worst' in the sense of the greatest overshoot of the 'turn-around' point. Only the large drops in atmospheric density associated with major solar storms in the 1981-1982 data result in overshoots beyond the leading/eastern boundary. Since the average atmospheric density was highest during this two year period, the average relative acceleration was highest and average time between maneuvers was shortest (about 8 days). This indicated that the western/trailing threshold and desired 'turn-around' point are promising. The approach meets the desired maneuver frequency without needing to separate the trailing/western threshold and the 'turn-around' point too much. This leaves adequate margin (200%) between the 'turn-around' point and the leading/eastern boundary.

Figure 9 shows the atmospheric density drop and corresponding overshoot in late 1981. One can see the sensitivity of the process to the timing of the maneuvers relative to a sharp drop in density. There are similar density drops beginning around days 315 and 345. The timing of the maneuvers for the first drop are such that the impact is spread across two maneuvers. In the second, case the density drop occurs just after the maneuver is calculated so the estimated relative acceleration is much higher than the actual relative acceleration, causing the overshoot.

Next, uncertainty in the relative locations, along with uncertainty in the relative velocity changes due to thruster variation were added. Also, the leading/eastern threshold to trigger braking maneuvers was included.

Calculating the relative location could also be thought of as subtracting the GPS locations of CloudSat and Picasso/Cena. The horizontal uncertainty from the Motorola Viceroy GPS receiver on CloudSat is 9 meters (1-sigma) with selective availability (SA) off. Assuming the same uncertainty in the Picasso/Cena receiver means the uncertainty in their difference is $9 * \sqrt{2}$ meters. Random Gaussian noise with this standard deviation was added to the 'true' relative locations. These noisy relative locations are then the basic inputs used to estimate CloudSat's velocity and acceleration relative to the 'ideal' point.

The leading threshold needs to be close enough to the leading boundary to not generate too many unnecessary braking maneuvers. However, it should not so be close that CloudSat will overshoot the leading boundary due to errors in the estimation of the relative velocity and acceleration and changes in the atmosphere in the two days between the GPS data collection and the maneuver. A leading threshold 14 km from the 'ideal' location toward the eastern boundary worked well for this study.

To estimate the relative velocity and acceleration (Step 3) at each time point, a parabola was fit through that relative location, the relative location one-half day earlier and the relative location one day earlier. This method would on rare occasions during low-density periods result in an acceleration estimate which was in the wrong direction (toward Picasso/Cena) due to the noisy data. To avoid this problem, the estimates over the previous day were averaged together. This somewhat crude estimate could be improved by fitting the parabola to more points. However, it is a simple calculation that is adequate for this problem, and one of the objectives of this analysis is to demonstrate that the maneuvers can be computed quite simply from the GPS data.

If a maneuver is conducted in the two days prior to the time of the estimate, it disturbs the parabolic motion and invalidates the estimation described above. To deal with this, pseudo-locations were created from which the impact of the requested maneuvers was removed. The parabolic fits were done using these psuedo-locations. If no maneuver had been conducted in the two days prior to the estimate the velocities from the parabolic fits over the prior day were averaged giving an estimate of the relative velocity one-half day earlier. One-half day multiplied by the estimated acceleration was then added to provide a current velocity estimate.

These velocity and acceleration estimates were then used to project the relative location ahead two days and that result was compared to the thresholds. If the estimate showed a crossing of the trailing threshold a maneuver was scheduled at the estimated crossing time. The estimated relative velocity going into the maneuver in two days is

$$\bar{v}_{in} = \bar{v}_r + 2 * \bar{a}_r \qquad (11)$$

where $\bar{v}_r$ and $\bar{a}_r$ are the estimated relative velocity and acceleration in km/day and km/day**2 respectively. The desired relative velocity coming out of the maneuver to achieve the desired 'turn-around' location for the this estimated relative acceleration is

Performance with Braking Maneuvers

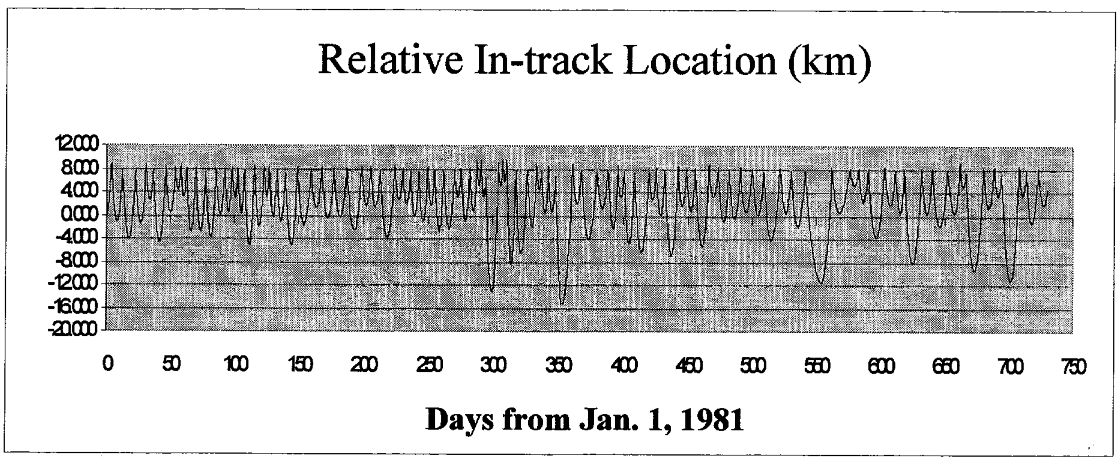



**Figure 10**

$$\bar{v}_{out} = -\sqrt{2d\bar{a}_r} \quad (12)$$

where $d$ is the distance between the estimated location two days ahead and the desired 'turn-around' point. Effectively, this is using the atmospheric density over that previous day (as reflected in the relative acceleration estimate) as an estimate for the atmospheric density for the days leading up to and following the maneuver.

Then the desired relative change in velocity, (not to be confused with $\Delta V$, the change in CloudSat's inertial velocity from the maneuver) is

$$\Delta v_{rel} = \bar{v}_{out} - \bar{v}_{in} \quad (13)$$

If the projected location two days ahead indicates a crossing of the leading/eastern threshold, the desired change in relative velocities is just enough to stop the estimated relative velocity so

$$\Delta v_{rel} = -\bar{v}_{in} \quad (14)$$

The uncertainty in thrust from CloudSat's thrusters is expected to be 2% (1 sigma). At the time of a maneuver this random error is added to the desired relative velocity change and the 'true' relative velocity is changed by that result. The actual $\Delta V$ of the maneuver would be 1/3 the relative velocity change and in the direction opposite the relative velocity change as discussed in Section 2.

Figure 10 shows a maneuver sequence using the 1981-1982 density data. Though this is the most challenging of the four data sets only two braking maneuvers were required around days 350 and 550 and in neither case was the leading boundary (-16 km in-track = -1 km cross track) exceeded.

Since the results are sensitive to the timing of the maneuvers relative to the changes in atmospheric density the starting location was varied from 0 to –12 km in .25 km intervals for each of the four two-year data sets generating 49 different maneuver sequences through each data set. The results of these runs are shown in Table 1. There are some interesting things to notice from this table. First, in all cases the average time between maneuvers meets the desire of 7 days or more. Second, there were a very few occasions when the leading boundary was overshot. However, there were very few braking maneuvers and the relative velocity change (and thus fuel consumption) due to these was a very small portion of the total relative velocity change. So, to avoid the overshoots one should probably pull the leading threshold back from the boundary to, say, - 8 km. This will result in a few additional braking maneuvers (1 to 3 per year), but they will have little impact on the total fuel budget, will prevent the rare overshoot, and leave some additional buffer to accommodate the effects of differences in the orbital elements of the two satellites which will be discussed in section 5

## 5. EQUATORIAL CROSSINGS VS. OTHER LATITUDES

This study has focused on relative motion of the two satellites due to the difference in their drag and the changing

**Table 1**

| | | Total # maneuvers | # Braking maneuvers | Lowest (Leading) Turnaround (km) | Highest (Trailing) Maneuver (km) | Total Relative Velocity Change (m/sec) | Braking Relative Velocity Change (m/sec) |
|---|---|---|---|---|---|---|---|
| 92-'93 | Average | 51.08 | 0.29 | -11.68 | 9.32 | 3.55 | 0.00 |
| | Max | 54 | 1.00 | -7.81 | 9.90 | 3.60 | 0.03 |
| | Min | 48 | 0.00 | -14.28 | 8.63 | 3.52 | 0.00 |
| | | | | | | | |
| 81-'82 | Average | 92.04 | 1.29 | -14.88 | 10.05 | 9.68 | 0.03 |
| | Max | 95 | 3.00 | -10.70 | 11.31 | 9.81 | 0.07 |
| | Min | 89 | 0.00 | -17.73 | 9.16 | 9.59 | 0.00 |
| | | | | | | | |
| 70-'71 | Average | 52.33 | 0.12 | -11.06 | 8.62 | 3.28 | 0.00 |
| | Max | 54 | 1.00 | -5.54 | 8.89 | 3.33 | 0.02 |
| | Min | 51 | 0.00 | -14.15 | 8.39 | 3.26 | 0.00 |
| | | | | | | | |
| 59-'60 | Average | 83.69 | 0.65 | -13.61 | 9.66 | 8.41 | 0.02 |
| | Max | 86 | 1.00 | -7.90 | 10.84 | 8.48 | 0.05 |
| | Min | 82 | 0.00 | -16.11 | 8.87 | 8.36 | 0.00 |

atmospheric density. The procedures described have established that by observing the relative locations of the satellite once per orbit at one of the equatorial crossings one should be able to keep that equatorial crossing point for CloudSat within +/- 1 km of Picasso/Cena's. As noted in section 2, the allowable in-track range increases by at least 1/cos(latitude) as the satellites move away from the equator. Intuitively small variations in eccentricity, and argument of perigee will not cause enough relative motion to exceed the in-track range at some other latitude before it is exceeded at the equator. If so, it would only be by some very small amount near the equator before the 1/cos(latitude) effect overcomes it. This has been confirmed using Aerospace Corporation's Satellite Orbit Analysis Program (SOAP). A difference of .0001 in eccentricity and 30° in argument of perigee (both larger than expected) results in a change in the in-track separation of about 9 km between the equator and the point of highest latitude (82°). However, the allowable in-track range relative to the 'ideal' point increases from +/- 16 km at the equator to +/- 32 km at 60° latitude and +/- 115 km at 82° latitude.

On the other hand, differences in the eccentricity and argument of perigee would mean the in-track separation of the two satellites would be different at the ascending and descending equatorial crossings. This is because even with the same semi-major axis (and therefore the same period) one satellite would have a slightly longer time between the ascending and descending equatorial crossings than the other. This would cause an offset in the relative location of CloudSat at the ascending crossing compared to that at the descending crossing. The in-track thresholds at the crossing where the calculations are performed might need to be reduced and or shifted somewhat to accommodate this difference. Based on the results in this study it looks a though a difference of 4 km or so in-track could be easily handled for these ballistic coefficients. Preliminary investigations using SOAP indicate that relative to the nominal Picasso orbit a maximum 4.5 km offset results from a 20 degree difference in argument of perigee or a .0001 difference in eccentricity.

Small variations in the inclination will cause the largest cross track separation to occur slightly off the equator. However, again based on SOAP, if the two ground tracks were separated at the equator by 1 km and the difference in their inclinations is the maximum specified variation of CloudSat's inclination of .00012°, their maximum ground track separation is only increased by 11 meters.

## 6. Conclusions and Further Efforts

This study has shown that under some simplifying assumptions it is possible to use a relatively simple curve fitting approach to calculate the timing and size of CloudSat's maneuvers based on data from GPS receivers on-board CloudSat and Picasso/Cena. The ground track of CloudSat can be maintained to within +/- 1 km in cross-track relative to Picasso/Cena's ground track. The average time between maneuvers is a week or more for the current estimates of the ballistic coefficients of the satellites. The implementation of the strategy does not require atmospheric prediction or orbit propagation models.

Throughout this paper it has been assumed that since the satellites are in nearly identical orbits they experience nearly identical orbit perturbations except for the difference in atmospheric drag. It is also assumed that the orbits are close enough to circular that any relative motion during an orbit, other than that due to the atmospheric drag, is small enough that one only needs to monitor the satellites at an equatorial crossing in order to assure the cross-track requirement is met throughout the orbit. However, these assumptions need to be confirmed against high-fidelity models. There are plans to test this strategy against high fidelity orbital simulations at both JPL and The Aerospace Corporation. These simulations include realistic models of the atmospheric drag, gravity, solar pressure, GPS, and spacecraft thrusters. In addition, these simulations will be used to determine the appropriate location on the orbit to conduct the maneuvers so that they do not, over the course of several maneuvers, create unacceptable differences in the eccentricity or argument of perigee of CloudSat relative to Picasso. It is believed that by conducting each maneuver near apogee or perigee, whichever reduces any eccentricity difference between the satellites, will do the job, but this needs further study.

If these tests confirm the reasonableness of the simplifying assumptions, the spreadsheet used in this analysis can be used to refine the strategy or quickly examine changes to the system such as the ballistic coefficients or the two day delay between GPS data collection and the maneuvers. For instance, since the relative location in the case presented doesn't approach the trailing boundary one might move the trailing maneuver threshold to 10 km or 12 km to accommodate differences in eccentricity and argument of perigee. This would result in slightly larger, less frequent maneuvers, but with a somewhat greater risk of overshoot. Pulling the leading threshold back to –8 km should catch the overshoots in time at the expense of a few more 'braking' maneuvers. The net impact may be fewer maneuvers overall at the expense of a slight increase in fuel consumption.

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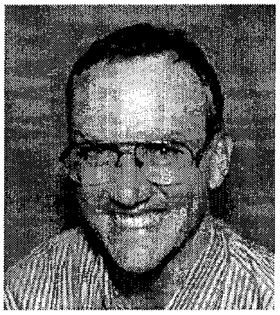

## APPENDIX

Consider a parabolic round trip path starting a distance $d_0 > 0$ from the 'ideal' location with a 'turn-around' location $d_T < d_0$ and a constant relative acceleration, $a_r$. Let $d(t)$ be the location at time t relative to the 'ideal' location and let the time $t = 0$ correspond to the time of the 'turn-around'. Then

$$d(t) = d_T + a_r * t^2 / 2 \qquad \text{(A1)}$$

and the relative velocity at time $t$ is

$$v_r(t) = a_r * t \qquad \text{(A2)}$$

If $d(t) = d_0$ in equation (A1), then $t = \pm\sqrt{2*(d_0 - d_T)/a_r}$ and the round trip time is

$$T_R = 2*\sqrt{2*(d_0 - d_T)/a_r} \qquad \text{(A3)}$$

Then from equation (A2) the relative velocities at the start and end of the roundtrip are

$$v_r(-T_R/2) = -\sqrt{2*(d_0 - d_T)*a_r} \qquad \text{(A4)}$$

and

$$v_r(T_R/2) = +\sqrt{2*(d_0 - d_T)*a_r} \qquad \text{(A5)}$$

The average distance from the 'ideal' point over the course of the roundtrip is

$$d_{ave} = (1/T_R)* \int_{-T_R/2}^{T_R/2} |d(t)| dt = (2/T_{rt})* \int_{0}^{T_R/2} |d(t)| dt \qquad \text{(A6)}$$

since $d(t)$ is symmetric about $t = 0$.

If $d_T \geq 0$,

$$\begin{aligned} d_{ave} &= (2/T_R)* \int_0^{T_R/2} (d_T + a_r * t^2/2) dt \\ &= (2/T_R)*\left[ \left(d_T * t + a_r * t^3/6\right)_0^{T_R/2} \right] \\ &= (2/T_R)*(T_R/2)*\left(d_T + a_r*(T_R/2)^2/6\right) \qquad \text{(A7)} \\ &= d_T + a_r*(T_R/2)^2/6 \\ &= d_T + a_r*(2*(d_0 - d_T)/a_r) \;\; /6 \\ &= (d_0 + 2*d_T)/3 \end{aligned}$$

If $d_T < 0$, then $d(t) < 0$ for $0 \le t < \sqrt{-2 * d_T / a_r}$, so

$$
\begin{aligned}
d_{ave} &= (2/T_R)*\left[\int_0^{\sqrt{-2*d_T/a_r}} -(d_T + a_r * t^2/2)dt + \int_{\sqrt{-2*d_T/a_r}}^{T_R/2} (d_T + a_r * t^2/2)dt\right] \\
&= (2/T_R)*\left[-\left(d_T * t + a_r * t^3/6\right)\Big|_0^{\sqrt{-2*d_T/a_r}} + \left(d_T * t + a_r * t^3/6\right)\Big|_{\sqrt{-2*d_T/a_r}}^{T_R/2}\right] \\
&= (2/T_R)*\left[(\sqrt{-2*d_T/a_r} * (-d_T + d_T/3)*2) + (T_R/2)*(d_T + (d_0 - d_T)/3)\right] \\
&= (-4*d_T/3)*\sqrt{-d_T/(d_0 - d_T)} + (d_0 + 2*d_T)/3 \\
&= (2*d_T + d_0 - 4*d_T*\sqrt{-d_T/(d_0 - d_T)})/3
\end{aligned}
\qquad (A8)
$$