A Comparison of Tolerance Analysis Methods

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We have seen many methods of calculating the worst case tolerance limits for electronic circuits. The intent of this paper is to demonstrate several different methods, and determine the results, and the corresponding confidence factors for each method.

The calculation methods addressed, and a brief description of each method is shown below:

1. Extreme Value Analysis - Each component is varied in the direction of the sensitivities to obtain the absolute worst case values of the circuit performance.

2. EVA Sensitivity Analysis - The parameter sensitivities are computed by evaluating the derivative of the output with respect to each component. The algebraic sum of the individual component tolerances (ie temperature, radiation, initial..) is multiplied by the sensitivity to determine the voltage variance for the component. The voltage variances of each part are summed algebraically to obtain the worst case circuit performance.

3. RSS Sensitivity 1 - The parameter sensitivities are computed at the nominal values. sum of each individual component tolerance (ie temperature, radiation, initial..) is multiplied by the sensitivity to determine the voltage variance for the component. The square root of the sum of the squares of each voltage variance is defined as the worst case circuit performance

4. RSS Sensitivity 2 - The parameter sensitivities are computed at the nominal values. The square root of the sum of the squares of each individual component tolerance (ie temperature, radiation, initial..) is multiplied by the sensitivity to determine the voltage variance for the component. The square root of the sum of the squares of each voltage variance is defined as the worst case circuit performance.

5. Monte Carlo - The component tolerances are algebraically added and entered into a SPICE simulator. The simulator randomly selects component values within the specified tolerance range, following a 12 point gaussian distribution. The results of the simulation include the population standard deviation, the population mean and normally, the 3 sigma limits for the worst case circuit performance.

A simple circuit was selected to apply each of these methods to The confidence level of each approach is defined later in this article in order to compare the results from each method.

The circuit selected for this example is an LM117 linear regulator circuit. The schematic of the circuit is shown in figure 1.

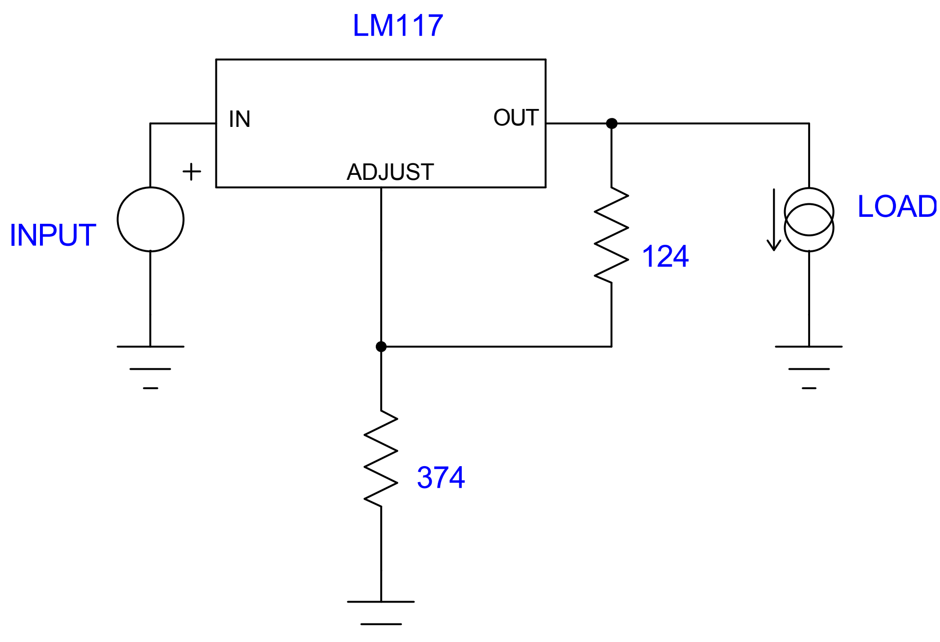


Figure 1 - Simple Evaluation Circuit

For this simple case the following symmetrical tolerances are defined for each part:

Component Tolerances

Part		initial	temp	age	Total
R1	124	1.00%	0.19	0.05	1.24%
R2	374	1.00%	0.19	0.05	1.24%
LM117 Rout	.00625	100.00%	0	0	100.00%
LM117 Ref	1.25	4.00%	2	2.64	8.64%
LM117 Iadj	55 uA	7.00E-05	0	0	7.00E-05
Load Current	0.75A	100.00%	0	0	100.00%

Extreme Value Analysis

nominal voltage and sensitivity calculations

$R2 := 374$ $R1 := 124$ $Vref := 1.25$ $Iadj := 55\cdot10^{-6}$ $R_o := .00625$ $I_o := 0.75$

$$V_{out} := \left[\left[\left[Vref - \left(I_o\cdot R_o\right)\right]\cdot\left(1 + \frac{R2}{R1}\right)\right]\right] + Iadj\cdot R2$$

$V_{out} = 5.022$

$dIadj := R2$

$dIadj = 374$

$$dR1 := -\left(Vref - I_o\cdot R_o\right)\cdot\frac{R2}{R1^2}$$

$dR1 = -0.03$

$$dR2 := \frac{\left(Vref - I_o\cdot R_o\right)}{R1} + Iadj$$

$dR2 = 0.01$

$$dRo := -I_o\cdot\left(1 + \frac{R2}{R1}\right)$$

$dRo = -3.012$

$$dIo := -R_o\cdot\left(1 + \frac{R2}{R1}\right)$$

$dIo = -0.025$

$$dVref := 1 + \frac{R2}{R1}$$

$dVref = 4.016$

Extreme Value Worst Case Maximum and Minimum Voltages

Maximum Voltage

$R2 := 374 \cdot \left(1 + \frac{1.24}{100}\right)$ $Vref := 1.25 \cdot \left(1 + \frac{8.64}{100}\right)$ $Iadj := (55 + 70) \cdot 10^{-6}$

$R1 := 124 \cdot \left(1 - \frac{1.24}{100}\right)$ $R_o := .00625 \cdot \left(1 - \frac{100}{100}\right)$ $I_o := 0.75 \cdot \left(1 - \frac{100}{100}\right)$

$$V_{out} := \left[\left[\left[Vref - \left(I_o \cdot R_o\right)\right] \cdot \left(1 + \frac{R2}{R1}\right)\right]\right] + Iadj \cdot R2$$

Extreme Value Minimumoutput voltage $V_{out} = 5.604$

Minimum Voltage

$R2 := 374 \cdot \left(1 - \frac{1.24}{100}\right)$ $Vref := 1.25 \cdot \left(1 - \frac{8.64}{100}\right)$ $Iadj := (55 - 70) \cdot 10^{-6}$

$R1 := 124 \cdot \left(1 + \frac{1.24}{100}\right)$ $R_o := .00625 \cdot \left(1 + \frac{100}{100}\right)$ $I_o := 0.75 \cdot \left(1 + \frac{100}{100}\right)$

$$Vout := \left[\left[\left[Vref - \left(I_o \cdot R_o\right)\right] \cdot \left(1 + \frac{R2}{R1}\right)\right]\right] + Iadj \cdot R2$$

Extreme Value Minimumoutput voltage $Vout = 4.423$

EVA Sensitivity Analysis

Part	Value	Sensitivity	Relative	initial	temp	age	Tol	Abs Value
R1	1.24E+02	-3.03E-02	-3.76E-02	1.00%	0.19	0.05	1.24%	0.047
R2	3.74E+02	1.01E-02	3.78E-02	1.00%	0.19	0.05	1.24%	0.047
ROUT	6.25E-03	-3.05E+00	-1.91E-04	100.00%	0	0	100.00%	0.019
VREF	1.25E+00	4.02E+00	5.02E-02	4.00%	2	2.64	8.64%	0.434
IADJ	5.50E-05	3.74E+02	2.06E-04	7.00E-05	0	0	7.00E-05	0.026
ILOAD	7.50E-01	-2.51E-02	-1.88E-04	100.00%	0	0	100.00%	0.019
Vnominal	5.023				EVA Tol	0.591	Volts	

RSS1 Sensitivity Analysis

Part	Value	Sensitivity	Relative	initial	temp	age	Tol	Abs Value
R1	1.24E+02	-3.03E-02	-3.76E-02	1.00%	0.19	0.05	1.24%	0.047
R2	3.74E+02	1.01E-02	3.78E-02	1.00%	0.19	0.05	1.24%	0.047
ROUT	6.25E-03	-3.05E+00	-1.91E-04	100.00%	0	0	100.00%	0.019
VREF	1.25E+00	4.02E+00	5.02E-02	4.00%	2	2.64	8.64%	0.434
IADJ	5.50E-05	3.74E+02	2.06E-04	7.00E-05	0	0	7.00E-05	0.026
ILOAD	7.50E-01	-2.51E-02	-1.88E-04	100.00%	0	0	100.00%	0.019
Vnominal	5.023							
					RSS Tol	0.440	Volts	

RSS2 Sensitivity Analysis

Part	Value	Sensitivity	Relative	initial	temp	age	RSS Tol	RSS Row
R1	1.24E+02	-3.03E-02	-3.76E-02	1.00%	0.19	0.05	1.02%	3.83E-02
R2	3.74E+02	1.01E-02	3.78E-02	1.00%	0.19	0.05	1.02%	3.85E-02
ROUT	6.25E-03	-3.05E+00	-1.91E-04	100.00%	0	0	100.00%	1.91E-02
VREF	1.25E+00	4.02E+00	5.02E-02	4.00%	2	2.64	5.19%	2.61E-01
IADJ	5.50E-05	3.74E+02	2.06E-04	7.00E-05	0	0	7.00E-05	2.62E-02
ILOAD	7.50E-01	-2.51E-02	-1.88E-04	100.00%	0	0	100.00%	1.88E-02
Vnominal	5.023				RSS2 Tol	0.268973	Volts	

SPICE Monte Carlo Analysis

The SPICE circuit for this circuit is shown in figure 2.

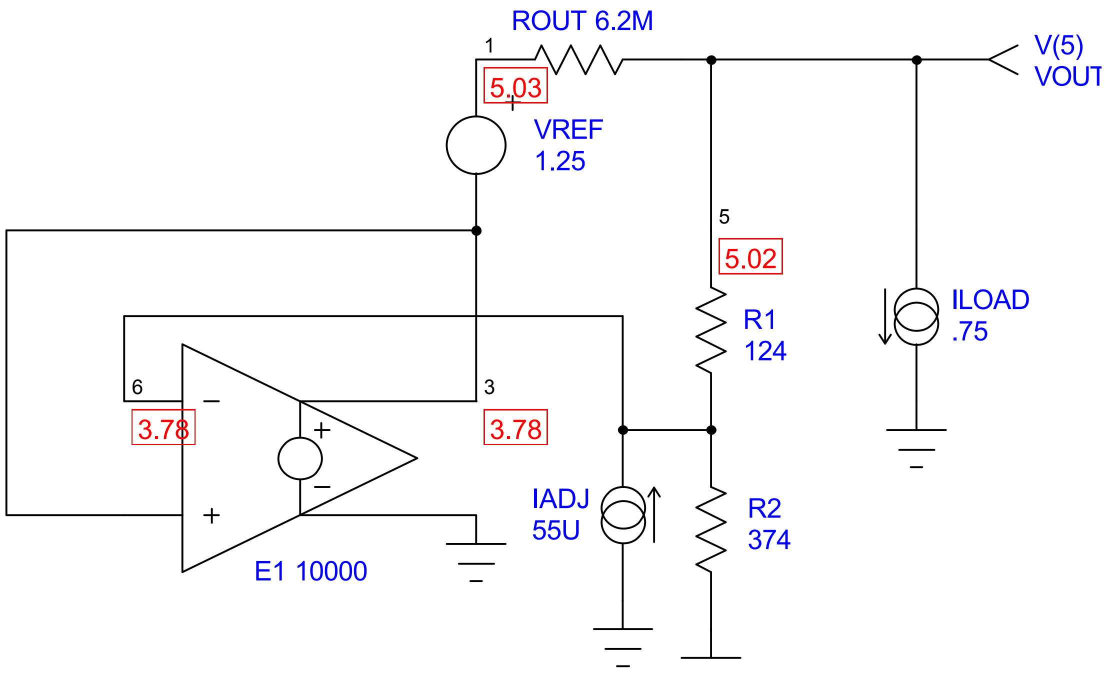


Figure 2 Spice Schematic

Spice Monte Carlo Netlist

```
F:\TEMP\lm117
.OP
.TRAN 1U 100U
.PRINT TRAN V(5)
*ALIAS  V(5)=VOUT
VREF 1 3 1.25 TOL=8.64%
RTOP 5 6 124 TOL=1.24%
RBOT 6 0 374 TOL=1.24%
IADJ 0 6 55U TOL=70U
ROUT 1 5 6.25M TOL=6.25M
ILOAD 5 0 .75 TOL=.75
E1 3 0 3 6 10000
.END
```

Monte Carlo Results

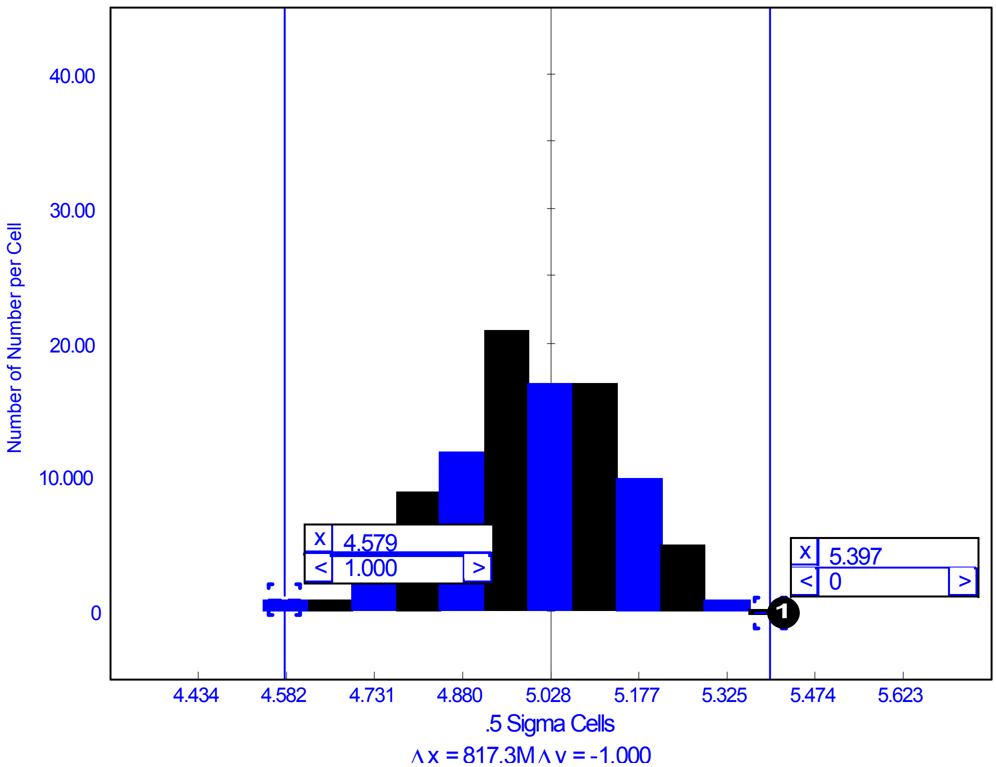
The results of 100 cases, performed in a Monte Carlo SPICE simulation, are shown below.

1	5.1924
2	5.0099
3	5.107
4	5.1488
5	5.0898
6	4.9969
7	5.0128
8	5.3198
9	4.8813
10	5.099
11	5.1541
12	5.3301
13	4.915
14	5.1745
15	5.1916
16	4.9378
17	4.9538
18	4.7793
19	5.0798
20	5.2465
21	4.7666
22	5.056
23	5.056
24	5.2808
25	4.9699
26	4.9986
27	4.9667
28	4.9621
29	5.1424
30	5.0145
31	4.9698
32	5.0635
33	4.7392
34	4.6917
35	5.0959
36	5.0351
37	4.8466
38	4.987
39	5.207

40	5.1717
41	5.1344
42	4.9889
43	5.0089
44	4.8358
45	4.8783
46	4.9972
47	5.0934
48	4.8432
49	5.1633
50	5.0917
51	4.7834
52	4.9136
53	4.8822
54	4.9711
55	5.1044
56	5.0693
57	5.0622
58	5.3137
59	4.8554
60	5.1819
61	4.9533
62	5.1175
63	4.9699
64	5.0142
65	5.2482
66	5.2886
67	5.0802
68	4.9757
69	5.0127
70	4.9546
71	5.0153
72	5.1374
73	5.1161
74	5.0907
75	4.9393
76	5.1218
77	4.9501
78	5.1876

79	5.1839
80	5.0727
81	5.1752
82	4.8133
83	4.9281
84	5.0251
85	4.8478
86	4.8168
87	4.9089
88	5.1033
89	5.0903
90	4.9508
91	5.1078
92	4.7521
93	5.1305
94	4.8451
95	5.2143
96	4.9566
97	5.3247
98	4.758
99	5.1846
100	4.6512
Mean	5.032088
Pop Stdev	0.14441

Monte Carlo Histogram Results



Statistical Evaluation

The results of the Monte Carlo Analysis yield a population mean and a population standard deviation. In order to determine the Extreme Value Worst Case circuit performance we need to select a confidence level. Selecting a confidence level of 0.99998, meaning that we have 99.998 percent confidence that **any** device will remain within these limits, we can define the number of standard deviations from the mean. EXCEL was used to compute the confidence results.

The resulting number of standard deviations for a confidence level of 0.99998 is 4.265. The resulting minimum and maximum values can be computed as

$$V\min = mean - (4.265 * s)$$

$$V\max = mean + (4.265 * s)$$

The results of the Monte Carlo analysis provided a population mean of 5.032 volts with a population standard deviation of 0.14441 volts.

This results in a maximum of 5.648 volts and a minimum of 4.416 volts.

Comparative Results

The table below shows the mean, minimum, and maximum output voltages, the effective number of standard deviations from the mean and the respective confidence level.

Method	Mean	Minimum	Maximum	# of STDEV	Confidence
Extreme Value	5.013	4.423	5.604	4.414*	100%
EVA Sensitivity	5.023	4.432	5.614	4.092	99.996
RSS1	5.023	4.583	5.463	3.048	99.770%
RSS2	5.023	4.754	5.292	1.863	93.750%
Monte Carlo	5.032	4.416	5.648	4.265	99.998

* 4.414 yields a confidence of 99.999%

Conclusions

Different circuits will result in different tolerances. This paper merely demonstrates the relative performance of each of the methods. It does show that Monte Carlo may be a reasonable method of determining the Extreme Value performance, if it is combined with a confidence level for the result.